Easing Traffic

The Lincoln Tunnel Authority is about to complete a tunnel under the city centre. Owing to financial cutbacks the tunnel is only one lane in either direction. The traffic Manager has realised that there will be holdups at both ends of the tunnel during the morning and evening rush hours. Bearing in mind aspects of safety and the desire to produce the maximum flow of traffic at peak times, he wishes to put up signs indicating a maximum speed and the distance to be maintained between vehicles.

What recommendations would you make to the Traffic Manager on this matter?

Name: Kaz Erdos

In[]:=
```
reaction = {30, 50, 70};
vel = {30, 50, 70};
brake = {45, 125, 245};
dist = {75, 175, 315};
velVdis = Transpose[{vel, dist}];
Text[Grid[Prepend[velVdis , {"vel", "dist"}],
  Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]
```

Out[]=

vel	dist
30	75
50	175
70	315

In[]:=
```
lp1 = ListPlot[velVdis, AxesLabel → {"vel", "dist"},
   PlotLabel → Style["Easing Traffic", Blue, 16],
   PlotStyle → {Red, PointSize[0.015]}, PlotRange → {{0, 100}, {0, 350}},
   AxesStyle → Arrowheads[0.025], ImageSize → Large];
```

In[]:=
```
Fit[velVdis, {x, x^2}, x]
```

Out[]= $1. x + 0.05 x^2$

In[]:=
```
0.999999999999967` x + 0.05000000000000002` x^2
dist1[x_] := 0.999999999999967` x + 0.05000000000000002` x^2
dist1[30]
```

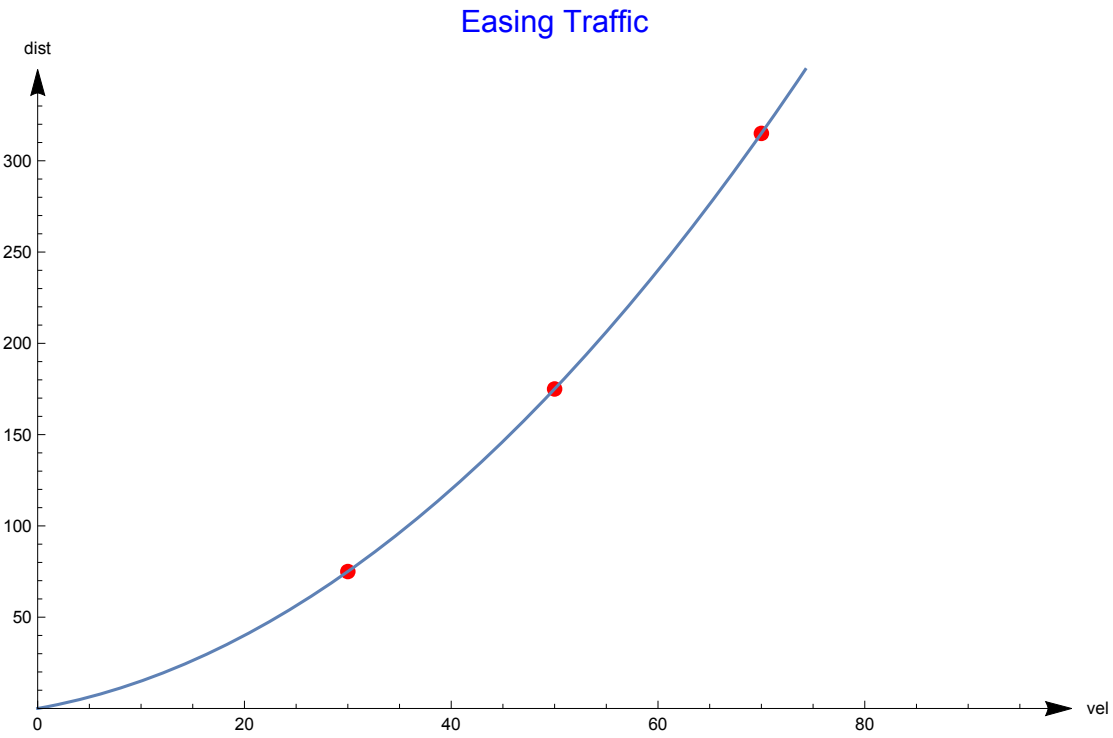
Out[]= $1. x + 0.05 x^2$

Out[]= 75.

In[•]:=
```
74.99999999999903`
plot1 = Plot[dist1[x], {x, 0, 100}, AxesLabel → {"velocity", "distance"},
   PlotLabel → Style["Easing Traffic", Blue, 16] ×
      PlotStyle → {Red, PointSize[0.015]}, PlotRange → {{0, 100}, {0, 350}},
   AxesStyle → Arrowheads[0.025], ImageSize → Large];
Show[
 lp1,
 plot1]
```

Out[•]=
```
75.
```

Out[•]=



In[•]:=
```
vel2 = {5, 10, 20, 30, 40, 50, 60, 70}
```

Out[•]=
```
{5, 10, 20, 30, 40, 50, 60, 70}
```

In[•]:=
```
dist2 = dist1[vel2]
```

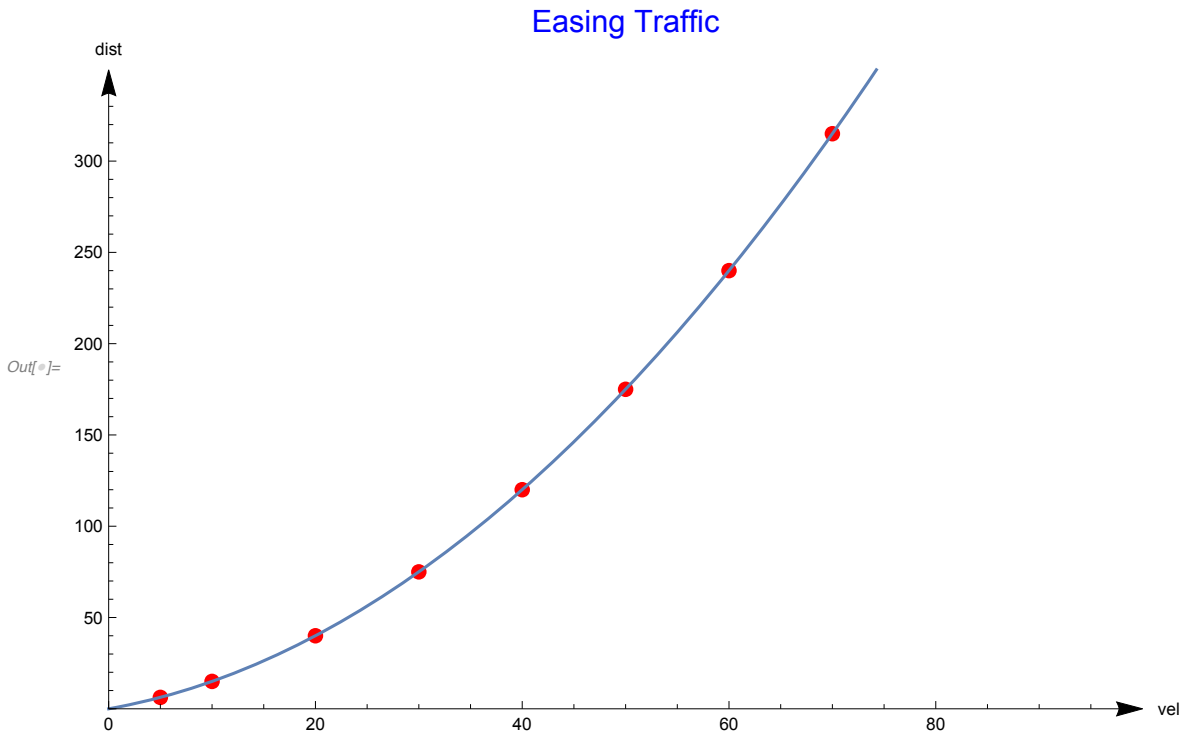
Out[•]=
```
{6.25, 15., 40., 75., 120., 175., 240., 315.}
```

In[•]:=
```
table2 = Transpose[{vel2, dist2}]
```

Out[•]=
```
{{5, 6.25}, {10, 15.}, {20, 40.}, {30, 75.},
 {40, 120.}, {50, 175.}, {60, 240.}, {70, 315.}}
```

```
In[•]:= lp2 = ListPlot[table2, AxesLabel → {"vel", "dist"},
           PlotLabel → Style["Easing Traffic", Blue, 16],
           PlotStyle → {Red, PointSize[0.015]}, PlotRange → {{0, 100}, {0, 350}},
           AxesStyle → Arrowheads[0.025], ImageSize → Large];
        Show[lp2, plot1]
```

Out[•]=



Now we have a generation equation for the overall stopping distance based on the travelling speed:
dist[v] = 1 * v + 0.05 * v^2, where dist is in feet and v is in mph.

Model 1

```
In[•]:= ncarsf[vel2_, dist2_] := (88 * vel2) / dist2

In[•]:= ncarsf[5, 6.25]
Out[•]= 70.4

In[•]:= ncars1 = Floor[ncarsf[vel2, dist2]]
Out[•]= {70, 58, 44, 35, 29, 25, 22, 19}
```

```
In[•]:= table3 = Transpose[{vel2, ncars1}]
Out[•]= {{5, 70}, {10, 58}, {20, 44}, {30, 35}, {40, 29}, {50, 25}, {60, 22}, {70, 19}}

In[•]:= Text[Grid[Prepend[table2, {"vel2", "ncars1"}],
    Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]
```

vel2	ncars1
5	6.25
10	15.
20	40.
30	75.
40	120.
50	175.
60	240.
70	315.

Out[•]=

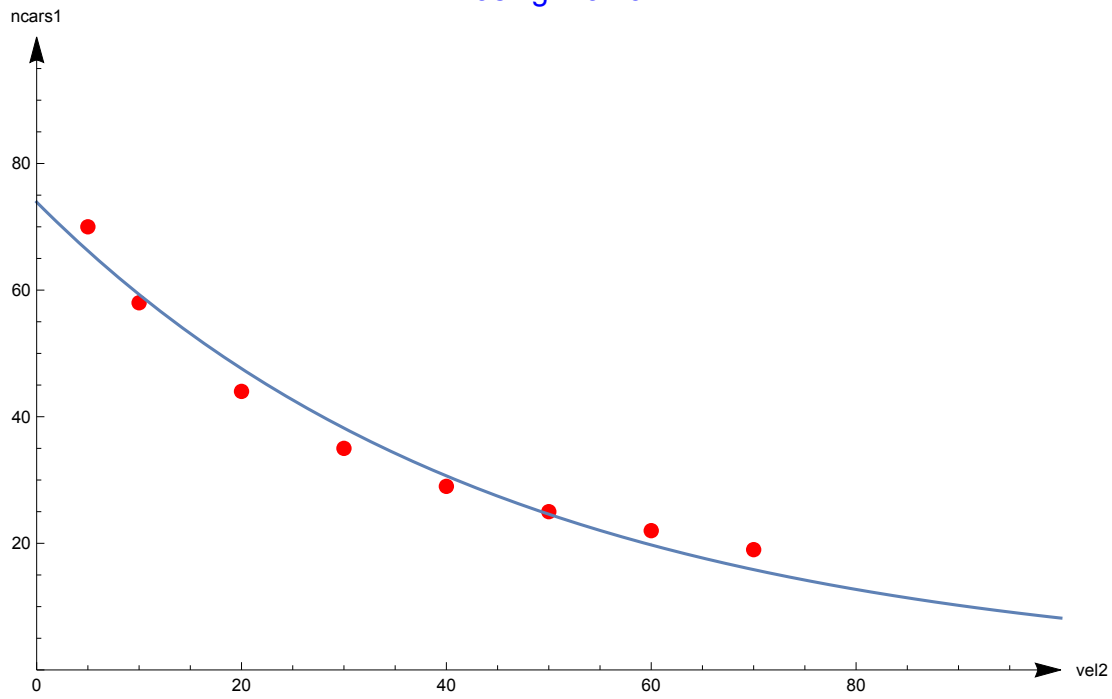
```
In[•]:= lp3 = ListPlot[table3, AxesLabel → {"vel2", "ncars1"},
     PlotLabel → Style["Easing Traffic", Blue, 16],
     PlotStyle → {Red, PointSize[0.015]}, PlotRange → {{0, 100}, {0, 100}},
     AxesStyle → Arrowheads[0.025], ImageSize → Large];
    model2 = a Exp[-k t];
    fit2 = FindFit[table2, model2, {a, k}, t]
Out[•]= {a → 24.7399, k → -0.0369883}

In[•]:= {a → 73.85527851781974`, k → 0.02222445901340004`}
    expfit[x_] := 73.9 * Exp[-0.022 x]
    expfit[1]
Out[•]= {a → 73.8553, k → 0.0222245}

Out[•]= 72.292

In[•]:= 72.29195337028443`
    plot3 = Plot[expfit[x], {x, 0, 100}, AxesLabel → {"ncars2", "dist"} ,
     PlotLabel → Style["Easing Traffic" , Blue, 18] ×
       PlotStyle → {Red, PointSize[0.015]}, PlotRange → {{0, 100}, {0, 100}},
     AxesStyle → Arrowheads[0.025], ImageSize → Large];
    Show[
     lp3,
     plot3]
Out[•]= 72.292
```

Out[•]=



In[•]:=
```
ncars2 = Floor[expfit[vel2]]
```

Out[•]=
```
{66, 59, 47, 38, 30, 24, 19, 15}
```

In[•]:=
```
table4 = Transpose[{vel2, ncars2}]
```

Out[•]=
```
{{5, 66}, {10, 59}, {20, 47}, {30, 38}, {40, 30}, {50, 24}, {60, 19}, {70, 15}}
```

In[•]:=
```
Text[Grid[Prepend[table4, {"vel2", "ncars2"}],
  Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]
```

Out[•]=

vel2	ncars2
5	66
10	59
20	47
30	38
40	30
50	24
60	19
70	15

For now, it seems that the maximum flow is 88 cars per minute at a speed of 0 miles per hour, but that does not make sense

Model 2

```
In[•]:= table5 = Transpose[{vel2, dist2, dist2 + 13}]
Out[•]= {{5, 6.25, 19.25}, {10, 15., 28.}, {20, 40., 53.}, {30, 75., 88.},
  {40, 120., 133.}, {50, 175., 188.}, {60, 240., 253.}, {70, 315., 328.}}

In[•]:= Text[Grid[Prepend[table5, {"vel2", "dist2", "dist2+13"}],
   Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {2, 2, 2}]]
```

vel2	dist2	dist2+13
5	6.25	19.25
10	15.	28.
20	40.	53.
30	75.	88.
40	120.	133.
50	175.	188.
60	240.	253.
70	315.	328.

```
In[•]:= ncars3 = Floor[ncarsf[vel2, dist2 + 13]]
Out[•]= {22, 31, 33, 30, 26, 23, 20, 18}

In[•]:= table6 = Transpose[{vel2, dist2, dist2 + 13, ncars3}]
Out[•]= {{5, 6.25, 19.25, 22}, {10, 15., 28., 31}, {20, 40., 53., 33}, {30, 75., 88., 30},
  {40, 120., 133., 26}, {50, 175., 188., 23}, {60, 240., 253., 20}, {70, 315., 328., 18}}

In[•]:= table7 = Transpose[{vel2, ncars3}]
Out[•]= {{5, 22}, {10, 31}, {20, 33}, {30, 30}, {40, 26}, {50, 23}, {60, 20}, {70, 18}}
```

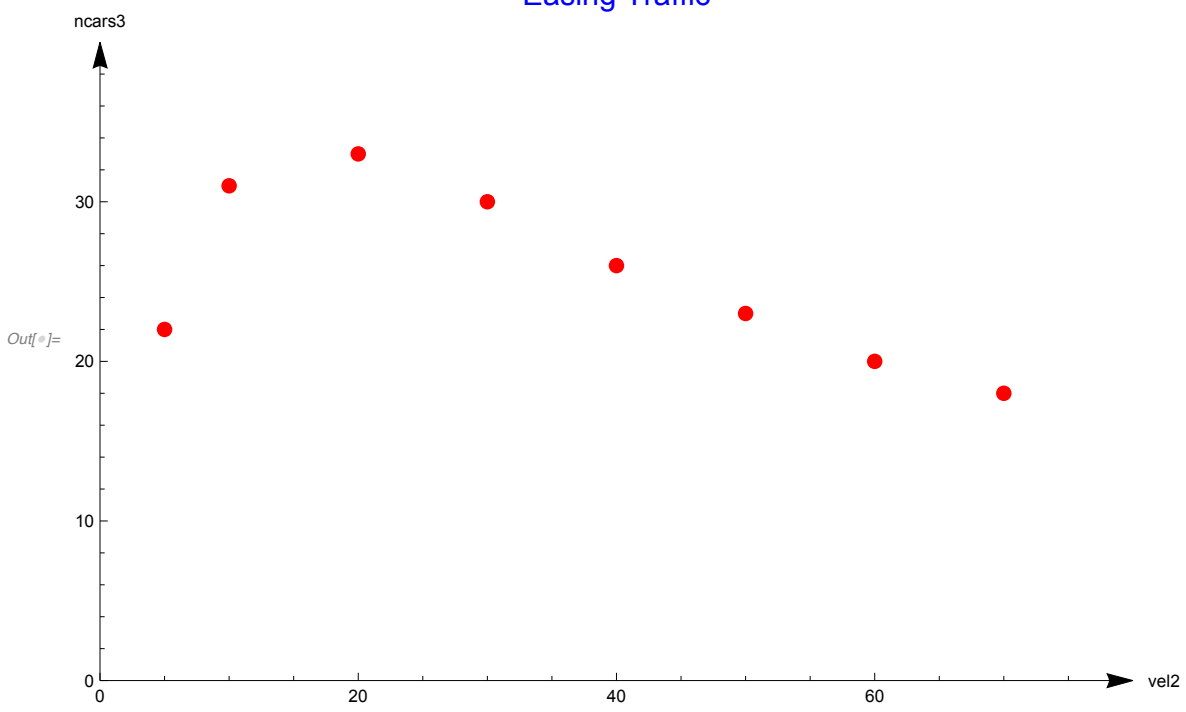
In[•]:=
```
Text[Grid[Prepend[table6, {"vel2", "dist2", "dist2+13", "ncars3"}],
  Alignment → Center, Dividers → {2 → True, 2 → True, 3 → Ture, 4 → True},
  Spacings → {2, 2, 2, 2}]]
```

Out[•]=

vel2	dist2	dist2+13	ncars3
5	6.25	19.25	22
10	15.	28.	31
20	40.	53.	33
30	75.	88.	30
40	120.	133.	26
50	175.	188.	23
60	240.	253.	20
70	315.	328.	18

In[•]:=
```
lp4 = ListPlot[table7, AxesLabel → {"vel2", "ncars3"},
  PlotLabel → Style["Easing Traffic", Blue, 16],
  PlotStyle → {Red, PointSize[0.015]}, PlotRange → {{0, 80}, {0, 40}},
  AxesStyle → Arrowheads[0.025], ImageSize → Large]
```

Out[•]=



Based on the plot above, the maximum number of cars exiting the tunnel is approximately 34 cars per

minute when the speed is about 15 miles per hor.

```
In[•]:= dist1[15]
Out[•]= 26.25
```

With an expected speed of 15 miles per hour, the expected separation between cars in the tunnel is 26 feet (which is approximately 2 car lengths).

Model 3

```
In[•]:= table8 = Transpose[{vel2, vel2, vel2 + 13}]

Out[•]= {{5, 5, 18}, {10, 10, 23}, {20, 20, 33}, {30, 30, 43},
  {40, 40, 53}, {50, 50, 63}, {60, 60, 73}, {70, 70, 83}}

In[•]:= Text[Grid[Prepend[table8, {"vel2", "dist2", "dist2+13"}],
    Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {2, 2, 2}]]
```

vel2	dist2	dist2+13
5	5	18
10	10	23
20	20	33
30	30	43
40	40	53
50	50	63
60	60	73
70	70	83

```
In[•]:= ncars4 = Floor[ncarsf[vel2, vel2 + 13]]
Out[•]= {24, 38, 53, 61, 66, 69, 72, 74}

In[•]:= table9 = Transpose[{vel2, vel2, vel2 + 13, ncars4}]
Out[•]= {{5, 5, 18, 24}, {10, 10, 23, 38}, {20, 20, 33, 53}, {30, 30, 43, 61},
  {40, 40, 53, 66}, {50, 50, 63, 69}, {60, 60, 73, 72}, {70, 70, 83, 74}}

In[•]:= table10 = Transpose[{vel2, ncars4}]
Out[•]= {{5, 24}, {10, 38}, {20, 53}, {30, 61}, {40, 66}, {50, 69}, {60, 72}, {70, 74}}
```

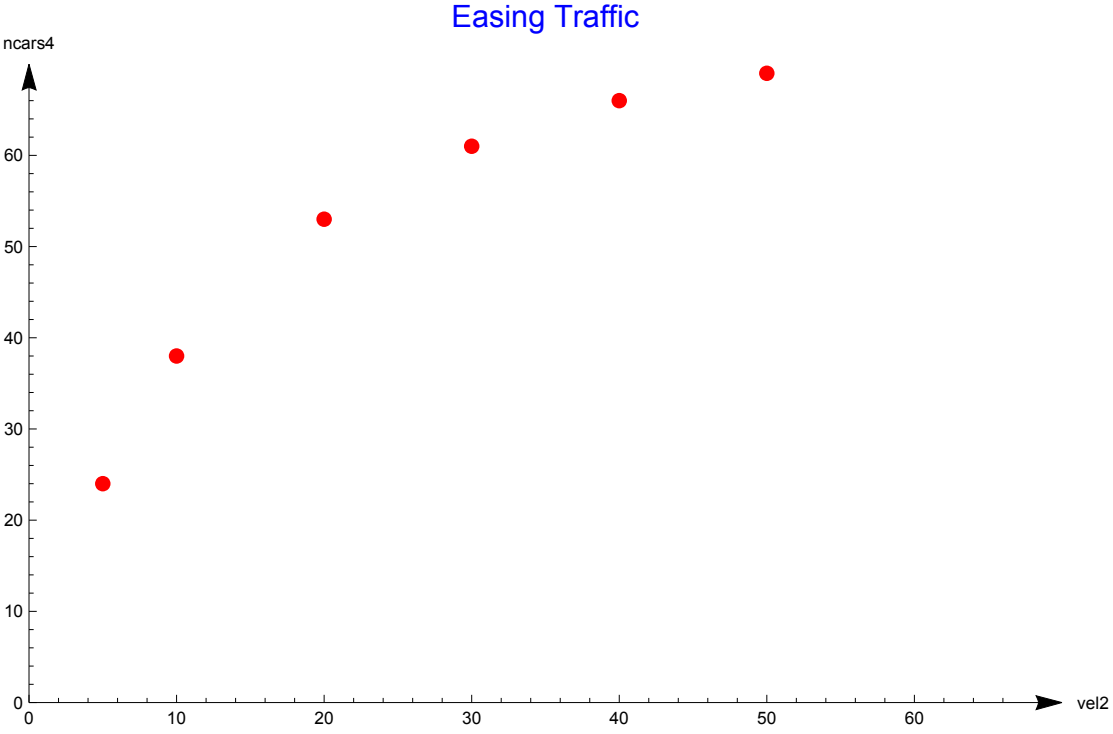
In[]:=
```
Text[Grid[Prepend[table9, {"vel2", "dist2", "dist2+13", "ncars4"}],
  Alignment → Center, Dividers → {2 → True, 2 → True, 3 → Ture, 4 → True},
  Spacings → {2, 2, 2, 2}]]
```

Out[]=

vel2	dist2	dist2+13	ncars4
5	5	18	24
10	10	23	38
20	20	33	53
30	30	43	61
40	40	53	66
50	50	63	69
60	60	73	72
70	70	83	74

In[]:=
```
lp4 = ListPlot[table10, AxesLabel → {"vel2", "ncars4"},
  PlotLabel → Style["Easing Traffic", Blue, 16],
  PlotStyle → {Red, PointSize[0.015]}, PlotRange → {{0, 70}, {0, 70}},
  AxesStyle → Arrowheads[0.025], ImageSize → Large]
```

Out[]=



We can see from the graph above that the maximum flow is found when the speed is 70mph (which is

the legal limit). The separation found is 70 feet, which is just over 5 car lengths. However, this would seem highly unsafe, so model 4 will serve as a compromise between models 2 and 3.

Model 4

```
In[•]:= sep[vel2_] := vel2 + (vel2^2) / 40
        dist3 = sep[vel2]
Out[•]= {45/8, 25/2, 30, 105/2, 80, 225/2, 150, 385/2}

In[•]:= table11 = Transpose[{vel2, dist3, dist3 + 13}]
Out[•]= {{5, 45/8, 149/8}, {10, 25/2, 51/2}, {20, 30, 43}, {30, 105/2, 131/2},
         {40, 80, 93}, {50, 225/2, 251/2}, {60, 150, 163}, {70, 385/2, 411/2}}

In[•]:= Text[Grid[Prepend[N[table11], {"vel2", "dist2", "dist2+13"}],
          Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {2, 2, 2}]]
```

vel2	dist2	dist2+13
5.	5.625	18.625
10.	12.5	25.5
20.	30.	43.
30.	52.5	65.5
40.	80.	93.
50.	112.5	125.5
60.	150.	163.
70.	192.5	205.5

```
In[•]:= ncars5 = Floor[ncarsf[vel2, dist3 + 13]]
        table12 = Transpose[{vel2, dist3, dist3 + 13, ncars5}]
Out[•]= {23, 34, 40, 40, 37, 35, 32, 29}

Out[•]= {{5, 45/8, 149/8, 23}, {10, 25/2, 51/2, 34}, {20, 30, 43, 40}, {30, 105/2, 131/2, 40},
         {40, 80, 93, 37}, {50, 225/2, 251/2, 35}, {60, 150, 163, 32}, {70, 385/2, 411/2, 29}}
```

In[•]:=
```
{{5, 45/8, 149/8, 24}, {10, 25/2, 51/2, 38}, {20, 30, 43, 53}, {30, 105/2, 131/2, 61},
 {40, 80, 93, 66}, {50, 225/2, 251/2, 69}, {60, 150, 163, 72}, {70, 385/2, 411/2, 74}}
```

Out[•]=
```
{{5, 45/8, 149/8, 24}, {10, 25/2, 51/2, 38}, {20, 30, 43, 53}, {30, 105/2, 131/2, 61},
 {40, 80, 93, 66}, {50, 225/2, 251/2, 69}, {60, 150, 163, 72}, {70, 385/2, 411/2, 74}}
```

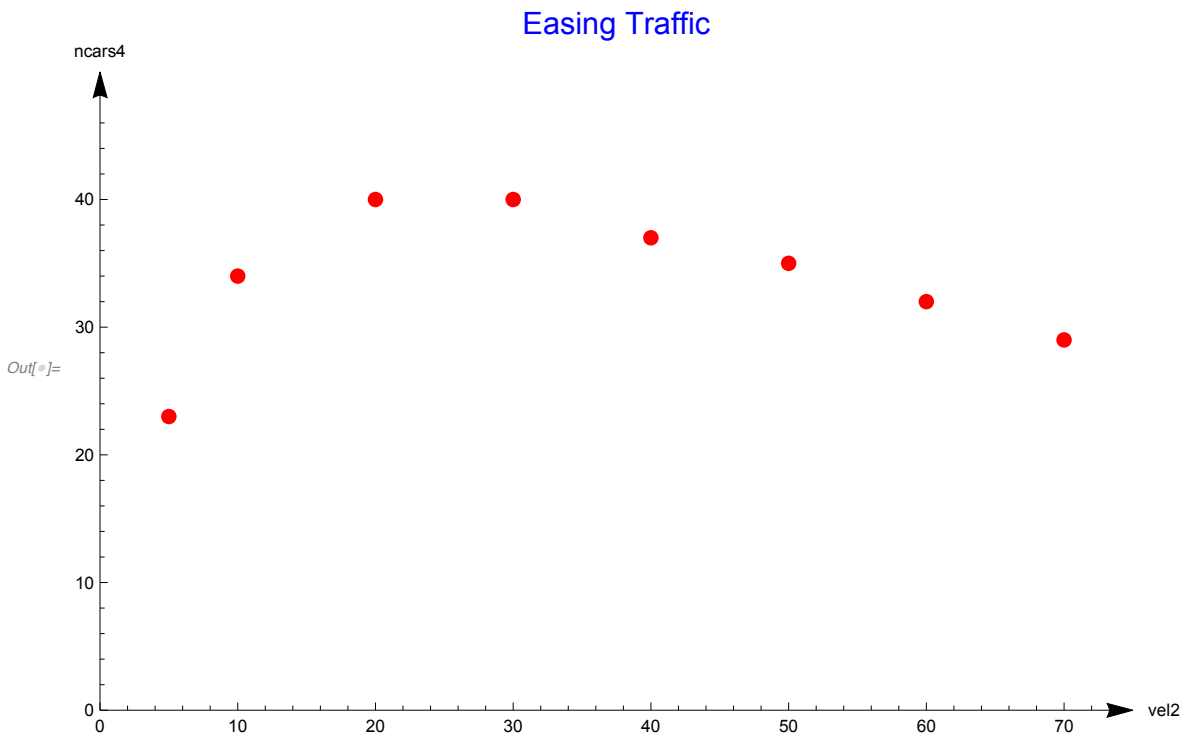
In[•]:=
```
Text[Grid[Prepend[N[table12], {"vel2", "dist2", "dist2+13", "ncars5"}],
  Alignment → Center, Dividers → {2 → True, 2 → True, 3 → Ture, 4 → True},
  Spacings → {2, 2, 2, 2}]]
```

Out[•]=

vel2	dist2	dist2+13	ncars5
5.	5.625	18.625	23.
10.	12.5	25.5	34.
20.	30.	43.	40.
30.	52.5	65.5	40.
40.	80.	93.	37.
50.	112.5	125.5	35.
60.	150.	163.	32.
70.	192.5	205.5	29.

In[•]:=
```
table13 = Transpose[{vel2, ncars5}];
lp5 = ListPlot[table13, AxesLabel → {"vel2", "ncars4"},
  PlotLabel → Style["Easing Traffic", Blue, 16],
  PlotStyle → {Red, PointSize[0.015]}, PlotRange → {{0, 75}, {0, 50}},
  AxesStyle → Arrowheads[0.025], ImageSize → Large]
```

Out[•]=



Based on this final graph, here is the final advice to the Traffic Manager. Based on the assumption that motorists often exceed the speed limit, it is advised that the speed limit posted should be 20mph with a separation between cars of 4 car lengths.

*In rush hours drive at 20mph with a separation of 4 car lengths.