Uber Pulley Calculus Problem

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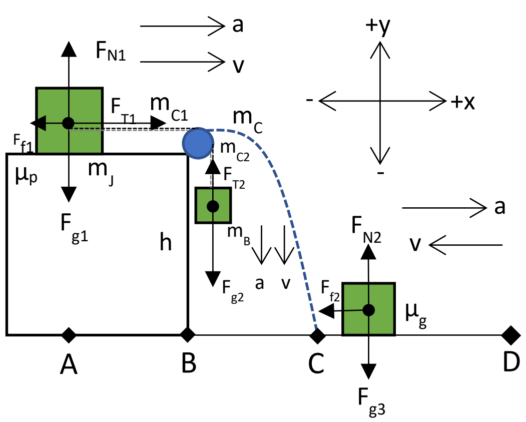
Description

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his (net) speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

Diagram



Givens

$$m_J = 60kg$$
$$m_B = 142kg$$
$$m_C = 54kg$$
$$L_C = 9m$$
$$h = 18m$$
$$\mu_P = 0.25$$

$$\Delta x_{BD} = 67m$$
$$x[t] = \frac{1}{2}at^2 + v_i t + x_i$$
$$v_f = v_i + a * t$$
$$v_f^2 = v_i^2 + 2a\Delta x$$

Assumptions

$$F_{T1} = F_{T2} \qquad a_1 = -a_2$$

Stage AB:

The first objective of this stage is calculating the initial velocity when Jerry flies off the platform. This can be calculated by working with sums of the forces that act on Jerry's jumper:

$$\sum F_y: F_{N1} - F_g = m_J * a_y, \ \ a_y = 0$$
$$F_{N1} = m_J * g$$
$$F_{N1} = 60 * 9.8$$
$$\underline{F_{N1} = 588N}$$

With the normal force, we can calculate friction:

$$F_{f1} = \mu_P * F_{N1}$$
$$F_{f1} = .25 * 588$$
$$\underline{F_{f1} = 147N}$$

With friction, we can now focus on the tension for this stage:

$$\sum F_x: F_{T1} - F_{f1} = m_{T1} * a_1$$
$$F_{T1} - 147 = m_{T1} * a_1$$
$$\underline{F_{T1} = m_{T1} * a_1 + 147}$$

As Jerry moves across the platform, mass is transferred from the horizontal part of the chain to the vertical part, since the chain itself has mass. Total Mass 1, as written above, refers to the combined mass of Jerry's jumper and the portion of the chain that is pulling him horizontally. The goal for the next step is to write mass in terms of the position of the chain, which can be up to 9 meters. In that case, the entire chain would be hanging off the platform. Then, acceleration can be written as a function of the chain's position, which can then be manipulated to find the velocity, as desired.

$$m_{T1}[x] = m_J + m_{C1}[x]$$

Mass of Chain 1 as a function of x refers to the mass of the horizontal component of the chain in relation to its position.

$$m_{C1}[x] = m_C - \frac{x}{l_C} * m_C$$
$$m_{C1}[x] = 54 - \frac{x}{9} * 54$$
$$\underline{m_{T1}[x] = 60 + (54 - \frac{x}{9} * 54)}$$

Now, we can focus on Total Mass 2, which refers to the combined mass of object B and the vertical portion of the chain.

$$m_{T2}[x] = m_B + m_{C2}[x]$$
$$m_{C2}[x] = \frac{x}{l_C} * m_C$$
$$\underline{m_{T2}[x] = 142 + \frac{x}{9} * 54}$$

Now refer to the sum of forces for object B and the vertical chain:

$$\sum F_y: F_{T2} - F_{g2} = m_{T2} * a_2$$
$$F_{T2} - m_{T2} * g = m_{T2} * a_2$$
$$\underline{F_{T2} = m_{T2} * g + m_{T2} * a_2}$$

$$\boldsymbol{Now\ Substitute\ F_{T1} = F_{T2}}$$

$$m_{T1} * a_1 + 147 = m_{T2} * g + m_{T2} * a_2$$
$$m_{T1} * a_1 + 147 = m_{T2} * g + m_{T2} * (-a_1)$$
$$a_1(m_{T1} + m_{T2}) = -147 + 9.8 * m_{T2}$$
$$a_1 = \frac{-147 + 9.8 * m_{T2}}{(m_{T1} + m_{T2})}$$
$$a_{1[x]} = \frac{-147 + 9.8 * (142 + \frac{x}{9} * 54)}{(60 + (54 - \frac{x}{9} * 54) + (142 + \frac{x}{9} * 54))}$$
$$a_{1[x]} = \frac{1244.6 + 58.8x}{256}$$
$$\underline{a_{1[x]} = 0.229688x + 4.86172}$$

Now, we can find velocity as a function of x based on this function:

$$a \equiv \frac{dv}{dt}$$

$$a = \frac{dx}{dx} * \frac{dv}{dt}$$

$$a = \frac{dx}{dt} * \frac{dv}{dt}$$

$$a = v * \frac{dv}{dt}$$

$$a[x]dx = v * dv$$

$$\int_{x_0}^{x} a[x]dx = \int_{v_0}^{v} v\, dv$$

$$\int_{x_0}^{x} 0.229688x + 4.86172\, dx = \frac{1}{2}v^2, TI\ NSpire$$

$$0.114826x^2 + 4.86172x = \frac{1}{2}v^2$$

$$0.229658x^2 + 9.72344x = v^2$$

$$\underline{v[x] = \sqrt{0.229658x^2 + 9.72344x}}$$

With velocity as a function of time, we can substitute 9, or the full length of the chain for the position, since the chain is fully vertical at the time of the launch.

$$v[9] = \sqrt{0.229658(9)^2 + 9.72344(9)}$$

$$\underline{v_{Bx} = 10.3011m/s}$$

Stage BC

During this stage, I will use kinematics to find the net velocity at the point where Jerry reaches the ground.

$$\Delta y = \frac{1}{2}a_y t^2 + v_{By}t$$

$$y[t] = \frac{1}{2}a_y t^2 + v_{By}t + h,\ v_{By} = 0$$

$$y[t] = \frac{1}{2}(-9.8)t^2 + h$$

$$y[t] = -4.9t^2 + 18$$

Set the function equal to zero, which corresponds to Jerry hitting the ground to find the time spend in air.

$$0 = -4.9t^2 + 18$$

$$t^2 = 3.67347$$

$$\underline{\cancel{t = -1.91663s,} t = 1.91663s}$$

$$\Delta x = \frac{1}{2}a_x t^2 + v_{By}t$$

$$x[t] = v_{By}t, a_x = 0$$

$$x[1.91663] = 10.3011 * 1.91663$$

$$\underline{x_{BC} = 19.7434m}$$

$$v_{Cy} = v_{By} + a * t, v_{By} = 0$$

$$v_{Cy} = 0 - 9.8 * 1.91663$$

$$\underline{v_{Cy} = -18.783m/s}$$

To find the net velocity, we must consider both components of velocity and root the sum of their squares:

$$v_{net} = \sqrt{v_{cx}^2 + v_{cy}^2}$$

$$v_{net} = \sqrt{(10.3011)^2 + (-18.783)^2}$$

$$\underline{v_{net} = 21.4223m/s}$$

Stage CD

Now, as Jerry is on the ground, we can find the coefficient of friction. The jumper maintains 75% of its net speed upon impact, then travels across the ground with a negative acceleration due to friction. Using Newton's Second Law, we can calculate friction.

$$v_c = 0.75 * v_{net}$$

$$v_c = 0.75 * 21.4223$$

$$\underline{v_c = 16.0667m/s}$$

We can calculate acceleration by using velocity and some given displacement along with our calculated displacement:

$$v_d^2 = v_c^2 + 2a_x\Delta x, v_d = 0$$

$$0 = 16.0667^2 + 2a_x(67 - 19.7434)$$

$$-94.5132a_x = 258.139$$

$$\underline{a_x = -2.73125m/s^2}$$

$$\sum F_x: -F_f = m_J * a_x$$

$$-F_f = 60 * -2.73125$$

$$\underline{F_f = 163.875N}$$

$$\sum F_y: F_{N2} - F_{g3} = m_J * a_y, a_y = 0$$

$$F_{N2} = 60 * 9.8$$

$$\underline{F_{N2} = 588N}$$

$$F_f = \mu_G * F_{N2}$$

$$163.875 = \mu_G * 588$$

$$\mu_G = \frac{163.875}{588}$$

$$\boxed{\mu_G = 0.2787}$$